CHAPTER 10   Simplifying and Solving

Since the beginning of this course, you have studied several different types of equations and have developed successful methods to solve them. For example, you have learned how to solve linear equations, systems of linear equations, and quadratic equations.

In Chapter 10, you will **extend** your solving skills to include other types of equations, including equations with square roots, absolute values, and messy fractions.

Another focus of this chapter is on learning how to simplify algebraic fractions (called "rational expressions") and expressions with exponents. By using the special properties of the number 1 and the meaning of exponents, you will be able to simplify large, complicated expressions.

Guiding Questions

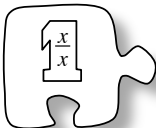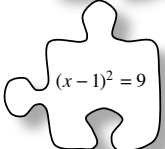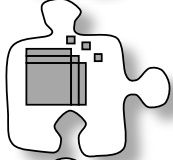Think about these questions throughout this chapter:

How can I rewrite it?
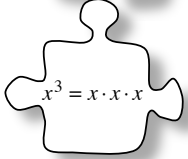
How can I solve it?

Is there another method?

What is special about the number 1?

In this chapter, you will learn how to:

➢ Simplify expressions involving exponents and fractions.

➢ Solve quadratic equations by completing the square.

➢ Use multiple methods to solve new types of equations and inequalities, such as those with square roots, rational expressions, and absolute values.

Chapter Outline

Section 10.1   In this section, you will study the properties of the number 1 and use them to simplify rational expressions and solve equations with fractions.

Section 10.2   Using the skills you learned in Section 10.1, you will develop new ways to solve unfamiliar, complicated equations involving square roots and absolute values.

Section 10.3   In this section, you will learn how to rewrite quadratics in perfect square form using a process called "completing the square."

Section 10.4   At the end of this chapter, you will use the meaning of an exponent to develop strategies to simplify exponential expressions.

                                                    Algebra Connections

10.1.1 How can I simplify?

Simplifying Expressions
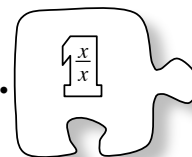
In Chapter 8, you used the special qualities of the number zero to develop a powerful way to solve factorable quadratics. In Section 10.1, you will focus on another important number: the number 1. What is special about 1? What can you do with the number 1 that you cannot do with any other number? You will use your understanding of the number 1 to simplify algebraic fractions, which are also known as **rational expressions**.

10-1. What do you know about the number 1? Brainstorm with your team and be ready to report your ideas to the class. Create examples to help show what you mean.

10-2. Mr. Wonder claims that anything divided by itself equals 1 (as long as you do not divide by zero). For example, he says that $\frac{16x}{16x} = 1$ if x is not zero.

    a. Is Mr. Wonder correct?

    b. Why can't x be zero?

    c. Next he considers $\frac{x-3}{x-3}$. Does this equal 1? What value of x must be excluded in this fraction?

    d. Create your own rational expression (algebraic fraction) that equals 1. **Justify** that it equals 1.

    e. Mr. Wonder also says that when you multiply any number by 1, the number stays the same. For example, he says that the product below equals $\frac{x}{y}$. Is he correct?

$$\frac{z}{z} \cdot \frac{x}{y} = \frac{x}{y}$$

10-3. Use what you know about the number 1 to simplify each expression below, if possible. State any values of the variables that would make the denominator zero.

    a. $\frac{x^2}{x^2}$

    b. $\frac{x}{x} \cdot \frac{x}{x} \cdot \frac{x}{3}$

    c. $\frac{x-2}{x-2} \cdot \frac{x+5}{x-1}$

    d. $\frac{9}{x} \cdot \frac{x}{9}$

    e. $\frac{h \cdot h \cdot k}{h}$

    f. $\frac{(2m-5)(m+6)}{(m+6)(3m+1)}$

    g. $\frac{6(n-2)^2}{3(n-2)}$

    h. $\frac{3-2x}{(4x-1)(3-2x)}$

10-4.   Mr. Wonder now tries to simplify $\frac{4x}{x}$ and $\frac{4+x}{x}$.

a.   Mr. Wonder thinks that since $\frac{x}{x}=1$, then $\frac{4x}{x}=4$.
Is he correct? Substitute three values of x to **justify**
your answer.

b.   He also wonders if $\frac{4+x}{x}=5$. Is this simplification correct?
Substitute three values of x to **justify** your answer. Remember
that $\frac{4+x}{x}$ is the same as $(4+x) \div x$.

c.   Compare the results of parts (a) and (b). When can a rational expression be
simplified in this manner?

d.   Which of the following expressions below is simplified correctly? Explain
how you know.

i. $\frac{x^2+x+3}{x+3}=x^2$ ii. $\frac{(x+2)(x+3)}{x+3}=x+2$

10-5.   In problem 10-4, you may have noticed that the numerator and denominator of an
algebraic fraction must both be written as a product before any terms create a 1.
Examine the expressions below. Factor the numerator and denominator of each
fraction, if necessary. That is, rewrite each one as a product. Then look for "ones"
and simplify. For each expression, assume the denominator is not zero.

a. $\frac{x^2+6x+9}{x^2-9}$ b. $\frac{2x^2-x-10}{3x^2+7x+2}$ c. $\frac{28x^2-x-15}{28x^2-x-15}$ d. $\frac{x^2+4x}{2x+8}$

10-6.   In your Learning Log, explain how to simplify rational expressions
such as those in problem 10-5. Be sure to include an example. Title
this entry "Simplifying Rational Expressions" and include today's date.

10-7.   How many solutions does each equation below have?

   a. $4x + 3 = 3x + 3$

   b. $3(x - 4) - x = 5 + 2x$

   c. $(5x - 2)(x + 4) = 0$

   d. $x^2 - 4x + 4 = 0$

10-8.   While David was solving the equation $100x + 300 = 500$, he wondered if he could first change the equation to $x + 3 = 5$. What do you think?

   a.   Solve both equations and verify that they have the same solution.

   b.   What could you do to the equation $100x + 300 = 500$ to change it into $x + 3 = 5$?

10-9.   Solve each of the following inequalities for the given variable. Represent your solutions on a number line.

   a. $5 + 3x < 5$

   b. $-3x \geq 8 - x$

10-10.   For each rational expression below, state any values of the variables that would make the denominator zero. Then complete each part.

   a.   Use the fact that $(x + 4)^2 = (x + 4)(x + 4)$ to rewrite $\frac{(x+4)^2}{(x+4)(x-2)}$. Then look for "ones" and simplify.

   b.   Use the strategy you used in part (a) to simplify the expression $\frac{8(x+2)^3(x-3)^3}{4(x+2)^2(x-3)^5}$.

10-11.   In Lesson 10.1.2 you will focus on multiplying and dividing rational expressions. Recall what you learned about multiplying and dividing fractions in a previous course as you answer the questions below. To help you, the following examples have been provided.

$$\frac{9}{16} \cdot \frac{4}{6} = \frac{36}{96} = \frac{3}{8}$$

$$\frac{5}{6} \div \frac{20}{12} = \frac{5}{6} \cdot \frac{12}{20} = \frac{60}{120} = \frac{1}{2}$$

   a.   Without a calculator, multiply $\frac{2}{3} \cdot \frac{9}{14}$ and reduce the result. Then use a calculator to check your answer. Describe your method for multiplying fractions.

   b.   Without a calculator, divide $\frac{3}{5} \div \frac{12}{25}$ and reduce the result. Then use a calculator to check your answer. Describe your method for dividing fractions.

10-12.   **Multiple Choice:** Which of the points below is a solution to $y < |x - 3|$?

   a.   (2, 1)   b.   (−4, 5)   c.   (−2, 8)   d.   (0, 3)

10.1.2 How can I rewrite it?

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Multiplying and Dividing Rational Expressions
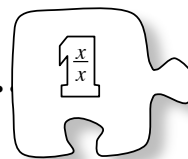
In a previous course you learned how to multiply and divide fractions. But what if the fractions have variables in them? (That is, what if they are rational expressions?) Is the process the same? Today you will learn how to multiply and divide rational expressions and will continue to practice simplifying rational expressions.

10-13. Review what you learned yesterday as you simplify the rational expression at right. What are the excluded values of x? (That is, what values can x <u>not</u> be?)

$$\frac{3x^2+11x-4}{2x^2+11x+12}$$

10-14. With your team, review your responses to homework problem 10-11. Verify that everyone obtained the same answers and be prepared to share with the class how you multiplied and divided the fractions below.

$$\frac{2}{3}\cdot\frac{9}{14} \qquad\qquad \frac{3}{5}\div\frac{12}{25}$$

10-15. Use your understanding of multiplying and dividing fractions to rewrite the expressions below. Then look for "ones" and simplify. For each rational expression, also state any values of the variables that would make the denominator zero.

a. $\dfrac{4x+3}{x-5}\cdot\dfrac{x-5}{x+3}$

b. $\dfrac{x+2}{9x-1}\div\dfrac{2x+1}{9x-1}$

c. $\dfrac{2m+3}{3m-2}\cdot\dfrac{7+4m}{3+2m}$

d. $\dfrac{(y-2)^3}{3y}\cdot\dfrac{y+5}{(y+2)(y-2)}$

e. $\dfrac{15x^3}{3y}\div\dfrac{10x^2y}{4y^2}$

f. $\dfrac{(5x-2)(3x+1)}{(2x-3)^2}\div\dfrac{(5x-2)(x-4)}{(x-4)(2x-3)}$

10-16. PUTTING IT ALL TOGETHER

Multiply or divide the expressions below. Leave your answers as simplified as possible. For each rational expression, assume the denominator is not zero.

a. $\dfrac{20}{22}\cdot\dfrac{14}{35}$

b. $\dfrac{12}{40}\div\dfrac{15}{6}$

c. $\dfrac{5x-15}{3x^2+10x-8}\div\dfrac{x^2+x-12}{3x^2-8x+4}$

d. $\dfrac{12x-18}{x^2-2x-15}\cdot\dfrac{x^2-x-12}{3x^2-9x-12}$

e. $\dfrac{5x^2+34x-7}{10x}\cdot\dfrac{5x}{x^2+4x-21}$

f. $\dfrac{2x^2+x-10}{x^2+2x-8}\div\dfrac{4x^2+20x+25}{x+4}$

10-17. In your Learning Log, explain how to multiply and divide rational expressions. Include an example of each. Title this entry "Multiplying and Dividing Rational Expressions" and include today's date.

$\mathbf{M}$ETHODS AND $\mathbf{M}$EANINGS

MATH NOTES

Rewriting Rational Expressions

To simplify a rational expression, both the numerator and denominator must be written in factored form. Then look for factors that make 1 and simplify. Study Examples 1 and 2 below.

Example 1: $\dfrac{x^2+5x+4}{x^2+x-12} = \dfrac{(x+4)(x+1)}{(x+4)(x-3)} = 1\cdot\dfrac{x+1}{x-3} = \dfrac{x+1}{x-3}$ for $x \neq -4$ or 3

Example 2: $\dfrac{2x-7}{2x^2+3x-35} = \dfrac{(2x-7)(1)}{(2x-7)(x+5)} = 1\cdot\dfrac{1}{x+5} = \dfrac{1}{x+5}$ for $x \neq -5$ or $\frac{7}{2}$

Just as you can multiply and divide fractions, you can multiply and divide rational expressions.

Example 3: Multiply $\dfrac{x^2+6x}{(x+6)^2} \cdot \dfrac{x^2+7x+6}{x^2-1}$ and simplify for $x \neq -6$ or 1.

After factoring, this expression becomes: $\dfrac{x(x+6)}{(x+6)(x+6)} \cdot \dfrac{(x+1)(x+6)}{(x+1)(x-1)}$

After multiplying, reorder the factors: $\dfrac{(x+6)}{(x+6)} \cdot \dfrac{(x+6)}{(x+6)} \cdot \dfrac{x}{(x-1)} \cdot \dfrac{(x+1)}{(x+1)}$

Since $\dfrac{(x+6)}{(x+6)} = 1$ and $\dfrac{(x+1)}{(x+1)} = 1$, simplify: $1 \cdot 1 \cdot \dfrac{x}{(x-1)} \cdot 1 \implies \dfrac{x}{(x-1)}$

Example 4: Divide $\dfrac{x^2-4x-5}{x^2-4x+4} \div \dfrac{x^2-2x-15}{x^2+4x-12}$ and simplify for $x \neq 2,\ 5,\ -3,$ or -6.

First, change to a multiplication expression: $\dfrac{x^2-4x-5}{x^2-4x+4} \cdot \dfrac{x^2+4x-12}{x^2-2x-15}$

Then factor each expression: $\dfrac{(x-5)(x+1)}{(x-2)(x-2)} \cdot \dfrac{(x-2)(x+6)}{(x-5)(x+3)}$

After multiplying, reorder the factors: $\dfrac{(x-5)}{(x-5)} \cdot \dfrac{(x-2)}{(x-2)} \cdot \dfrac{(x+1)}{(x-2)} \cdot \dfrac{(x+6)}{(x+3)}$

Since $\dfrac{(x-5)}{(x-5)} = 1$ and $\dfrac{(x-2)}{(x-2)} = 1$, simplify to get: $\dfrac{(x+1)(x+6)}{(x-2)(x+3)} \implies \dfrac{x^2+7x+6}{x^2+x-6}$

Note: From this point forward in the course, you may assume that all values of x that would make a denominator zero are excluded.

10-18.   Now David wants to solve the equation $4000x - 8000 = 16,000$.

   a.   What easier equation could he solve instead that would give him the same solution? (In other words, what equivalent equation has easier numbers to work with?)

   b.   **Justify** that your equation in part (a) is equivalent to $4000x - 8000 = 16,000$ by showing that they have the same solution.

   c.   David's last equation to solve is $\frac{x}{100} + \frac{3}{100} = \frac{8}{100}$. Write and solve an equivalent equation with easier numbers that would give him the same answer.

10-19.   Find the slope and *y*-intercept of each line below.

   a. $y = -\frac{6}{5}x - 7$ b. $3x - 2y = 10$

   c.   The line that goes through the points (5, –2) and (8, 4).

10-20.   Solve the systems of equations below using any method.

   a. $3x - 3 = y$ b. $3x - 2y = 30$
         $6x - 5y = 12$ $2x + 3y = -19$

10-21.   Simplify the expressions below.

   a. $\frac{x^2 - 8x + 16}{3x^2 - 10x - 8}$ for $x \neq -\frac{2}{3}$ or 4   b. $\frac{10x + 25}{2x^2 - x - 15}$ for $x \neq -\frac{5}{2}$ or 3

   c. $\frac{(k-4)(2k+1)}{5(2k+1)} \div \frac{(k-3)(k-4)}{10(k-3)}$ for $k \neq 3$, 4, or $-\frac{1}{2}$

10-22.   Solve the equations below. Check your solution(s).

   a. $\frac{m}{6} = \frac{m+1}{5}$ b. $\frac{3x-5}{2} = \frac{4x+1}{4}$ c. $\frac{8}{k} = \frac{14}{k+3}$ d. $\frac{x}{9} = 10$

10-23.   A piece of metal at 20°C is warmed at a steady rate of 2 degrees per minute. At the same time, another piece of metal at 240°C is cooled at a steady rate of 3 degrees per minute. After how many minutes is the temperature of each piece of metal the same? Explain how you found your answer.

10.1.3 How can I solve it?

Solving by Rewriting
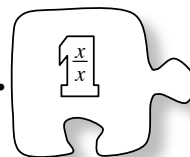
Lessons 10.1.1 and 10.1.2 focused on how to multiply, divide, and simplify rational expressions. How can you use these skills to solve problems?

10-24. Review what you learned in Lessons 10.1.1 and 10.1.2 by multiplying or dividing the expressions below. Simplify your results.

a. $\frac{x-7}{9(2x-1)} \div \frac{(x+5)(x-7)}{6x(x+5)}$

b. $\frac{6x^2-x-1}{3x^2+25x+8} \cdot \frac{x^2+4x-32}{2x^2+7x-4}$

10-25. Cassie wants to solve the quadratic equation $x^2 + 1.5x - 2.5 = 0$. "I think I need to use the Quadratic Formula because of the decimals," she told Claudia. Suddenly, Claudia blurted out, "No, Cassie! I think there is another way. Can't you first rewrite this equation so it has no decimals?"

a. What is Claudia talking about? Explain what she means. Then rewrite the equation so that it has no decimals.

b. Now solve the new equation (the one without decimals). Check your solution(s).

10-26. SOLVING BY REWRITING

Rewriting $x^2 + 1.5x - 2.5 = 0$ in problem 10-25 gave you a new, **equivalent** equation that was much easier to solve. If needed, refer to the Math Notes box for this lesson for more information about equivalent equations.

How can each equation below be rewritten so that it is easier to solve? With your team, find an equivalent equation for each equation below. Be sure your equivalent equation has no fractions or decimals and has numbers that are reasonably small. Strive to find the *simplest* equation. Then solve the new equation and check your answer(s).

a. $32(3x) - 32(5) = 32(7)$

b. $9000x^2 - 6000x - 15000 = 0$

c. $\frac{1}{3} + \frac{x}{3} = \frac{10}{3}$

d. $2x^2 + 4x - 2.5 = 0$

10-27.   Examine the equation below.

$$\frac{x}{6} - \frac{5}{8} = 4$$

a.   Multiply each term by 6. What happened? Do any fractions remain?

b.   If you have not already done so, decide how you can change your result from part (a) so that no fractions remain. Then solve the resulting equation.

c.   Multiplying $\frac{x}{6} - \frac{5}{8} = 4$ by 6 did not eliminate all the fractions. What could you have multiplied by to get rid of all the fractions? Explain how you got your answer and write the equivalent equation that has no fractions.

d.   Solve the resulting equation from part (c) and check your solution in the original equation.

10-28.   Now you are going to **reverse** the process. Your teacher will give your team a simple equation that you need to "complicate." Change the equation to make it seem harder (although you know it is still equivalent to the easy equation).

a.   Verify that your new equation is equivalent to the one assigned by your teacher.

b.   Share your new equation with the class by posting it on the overhead projector or chalkboard.

c.   Copy down the equations generated by your class on another piece of paper. You will need these equations for homework problem 10-29.

METHODS AND MEANINGS

MATH NOTES

Equivalent Equations

Two equations are **equivalent** if they have all the same solutions. There are many ways to change one equation into a different, equivalent equation. Common ways include: *adding* the same number to both sides, *subtracting* the same number from both sides, *multiplying* both sides by the same number, *dividing* both sides by the same (non-zero) number, and *rewriting* one or both sides of the equation.

For example, the equations below are all equivalent to $2x + 1 = 3$:

$$20x + 10 = 30 \qquad\qquad 2(x + 0.5) = 3$$

$$\frac{2x}{3} + \frac{1}{3} = 1 \qquad\qquad 0.002x + 0.001 = 0.003$$

10-29.   Solve the equations generated by your class in
problem 10-28. Be sure to check each solution
and show all work.

10-30.   Multiply or divide the expressions below.
Leave your answers as simplified as possible.

a. $\dfrac{(3x-1)(x+7)}{4(2x-5)} \cdot \dfrac{10(2x-5)}{(4x+1)(x+7)}$
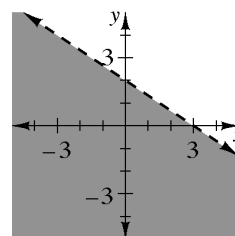
b. $\dfrac{(m-3)(m+11)}{(2m+5)(m-3)} \div \dfrac{(4m-3)(m+11)}{(4m-3)(2m+5)}$

c. $\dfrac{2p^2+5p-12}{2p^2-5p+3} \cdot \dfrac{p^2+8p-9}{3p^2+10p-8}$

d. $\dfrac{4x-12}{x^2+3x-10} \div \dfrac{2x^2-13x+21}{2x^2+3x-35}$

10-31.   Find the equation of the line parallel to $y=-\frac{1}{3}x+5$ that goes through the
point (9, –1).

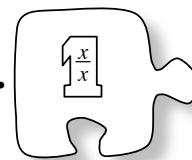10-32.   Write the inequality represented by the graph at right.

10-33.   Jessica has three fewer candies than twice the
number Dante has.

a.   If Dante has d candies, write an expression to represent how many candies
Jessica has.

b.   If Jessica has 19 candies, write and solve an equation to find out how many
candies Dante has.

10.1.4 How can I solve it?

Fraction Busters

In Lesson 10.1.3, you learned a powerful new method to help solve complicated equations: rewriting the equation first to create a simpler, equivalent equation. Today you will continue to solve new, complicated equations and will focus specifically on equations with fractions. As you solve these new problems, look for ways to **connect** today's work with what you have learned previously.

10-34. Examine the equation below.

$$\frac{5x}{3} + \frac{15}{2} = \frac{5}{2}$$

a. Solve the equation by first finding an equivalent equation without fractions. Check your solution(s).

b. Often, this method of eliminating fractions from an equation is called the **Fraction Busters Method** because the multiplication of the equation by a common denominator or several of the denominators eliminates ("busts") the fractions. The result is an equation with no fractions.

By what number (or numbers) did you multiply both sides of the equation in part (a) to eliminate the fractions? How did you choose that number? Is it the smallest number that would eliminate all of the fractions?

10-35. Work with your team to solve each of the equations below by first finding an equivalent equation that contains no fractions. Each problem presents new challenges and situations. Be ready to **justify** how you solved each problem and share why you did what you did with the class. Remember to check each solution.

a. $\frac{x}{4} - \frac{x}{6} = \frac{2}{3}$

b. $\frac{5}{x} - 2x = 3$

c. $\frac{-2x+1}{3} - \frac{x+3}{7} = 8$

d. $\frac{x+3}{x-2} + 2 = \frac{x+5}{x-2}$

10-36. Now examine the equation below.

$$\frac{4+p}{p^2+2p-8} + 3 = \frac{4}{p-2}$$

a. What values of p are not allowed? Show how you know.

b. Use your new skills to rewrite the equation above so that it has no fractions. Then solve the new equation. Check your solution(s). What happened?

10-37. Solve the equations below by first changing each equation to a simpler, equivalent equation. Check your solution(s).
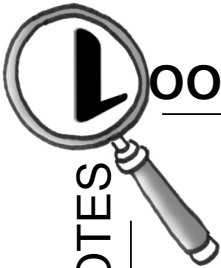
    a. $50x^2 + 200x = -150$

    b. $\frac{a}{9} + \frac{1}{a} = \frac{2}{3}$

    c. $1.2m - 0.2 = 3.8 + m$

    d. $\frac{2}{x+5} + \frac{3x}{x^2+2x-15} = \frac{4}{x-3}$

LOOKING DEEPER

MATH NOTES

Solving Equations with Algebraic Fractions
(also known as Fraction Busters)

Example: Solve $\frac{x}{3} + \frac{x}{5} = 2$ for x.

This equation would be much easier to solve if it had no fractions. Therefore, the first goal is to find an equivalent equation that has no fractions.

$$\frac{x}{3} + \frac{x}{5} = 2$$

The lowest common denominator of $\frac{x}{3}$ and $\frac{x}{5}$ is 15.

To eliminate the denominators, multiply both sides of the equation by the common denominator. In this example, the lowest common denominator is 15, so multiplying both sides of the equation by 15 eliminates the fractions. Another approach is to multiply both sides of the equation by one denominator and then by the other.

$$15 \cdot \left(\frac{x}{3} + \frac{x}{5}\right) = 15 \cdot 2$$

$$15 \cdot \frac{x}{3} + 15 \cdot \frac{x}{5} = 15 \cdot 2$$

Either way, the result is an equivalent equation without fractions:

$$5x + 3x = 30$$
$$8x = 30$$

The number used to eliminate the denominators is called a **fraction buster**. Now the equation looks like many you have seen before, and it can be solved in the usual way.

$$x = \frac{30}{8} = \frac{15}{4} = 3.75$$

$$\frac{3.75}{3} + \frac{3.75}{5} = 2$$

Once you have found the solution, remember to check your answer.

$$1.25 + 0.75 = 2$$

10-38. Solve the equations below by first changing each equation to a simpler equivalent equation. Check your solutions.

a. $3000x - 2000 = 10{,}000$

b. $\frac{x^2}{2} + \frac{3x}{2} - 5 = 0$

c. $\frac{5}{2}x - \frac{1}{3} = 13$

d. $\frac{3}{10} + \frac{2x}{5} = \frac{1}{2}$

10-39. Multiply or divide the expressions below. Express your answers as simply as possible.

a. $\frac{5x^2 - 11x + 2}{x^2 + 8x + 16} \cdot \frac{x^2 + 10x + 24}{10x^2 + 13x - 3}$

b. $\frac{6x + 3}{2x - 3} \div \frac{3x^2 - 12x - 15}{2x^2 - x - 3}$

10-40. To avoid a sand trap, a golfer hits a ball so that its height is represented by the equation $h = -16t^2 + 80t$, where h is the height measured in feet and t is the time measured in seconds.

a. When does the ball land on the ground?

b. What is the maximum height of the ball during its flight?

10-41. Write and solve an equation (or a system of equations) for the following situation. Be sure to define your variables.

Each morning, Jerry delivers two different newspapers: the *Times* and the *Star*. The *Times* weighs $\frac{1}{2}$ a pound and the *Star* weighs $\frac{1}{4}$ a pound. If he delivers a total of 27 newspapers that weigh a total of $11\frac{1}{2}$ pounds, how many *Times* newspapers does he deliver?

10-42. Graph the system of inequalities below on graph paper. Shade the region that represents the solution.

$$y \geq x^2 - 4$$
$$y \leq -x^2 + 4$$

10-43. **Multiple Choice:** $x = 2$ is a solution to which of the equations or inequalities below?

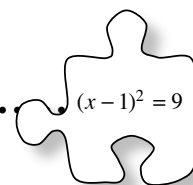a. $\frac{x-4}{3} = \frac{x}{15}$ b. $(x-2)^2 < 0$ c. $|3x - 8| \geq -1$ d. $\sqrt{x+2} = 16$

10.2.1 How can I solve it?

Multiple Methods for Solving Equations

$$(x-1)^2 = 9$$

So far in this course you have developed many different methods for solving equations, such as adding things to both sides of the equation or multiplying each term by a number to eliminate fractions. But how would you solve a complicated equation such as the one shown below?

$$(\sqrt{|x+5|} - 6)^2 + 4 = 20$$

By looking at equations in different ways, you will be able to solve some equations much more quickly and easily. These new approaches will also allow you to solve new kinds of equations you have not studied before. As you solve equations in today's lesson, ask your teammates these questions:

How can you see it?

Is there another way?

10-44.   DIFFERENT METHODS TO SOLVE AN EQUATION

By the end of this section you will be able to solve the equation $(\sqrt{|x+5|} - 6)^2 + 4 = 20$. This equation is very complex and will require you to look at solving equations in new ways. To be prepared for other strange and unfamiliar equations, you will first examine all of the solving tools you currently have by solving a comparatively easier equation:

$$4(x+3) = 20$$

Your Task: With your team, solve $4(x+3) = 20$ for x in *at least* two different ways. Explain how you found x in each case and be prepared to share your explanations with the class.

10-45.   SOLVING BY REWRITING

David wants to find x in the equation
$4(x+3) = 20$. He said, "*I can rewrite
this equation by distributing the 4 on the
left-hand side*." After distributing, what
should his new equation be? Solve this
equation using David's method.

10-46.   SOLVING BY UNDOING

Juan says, "I see the whole thing a different way." Here is how he explains his
approach to solving $4(x+3) = 20$, which he calls "undoing": "*Instead of
distributing first, I want to eliminate the 4 from the left side by undoing the
multiplication*."

a.   What can Juan do to both sides of the equation to remove the 4? Why does this
work?

b.   Solve the equation using Juan's method. Did you get the same result as David?

c.   Why is it appropriate for this method to be called "undoing"?

10-47.   SOLVING BY LOOKING INSIDE

Kenya said, "I solved David's equation in a much quicker way!" She solved the
equation $4(x+3) = 20$ with an approach that she calls "looking inside." Here is
how she described her thinking: "*I think about everything inside the parentheses as a
group. After all, the parentheses group all that stuff together. I think the contents of
the parentheses must be 5*."

a.   Why must the expression inside the parentheses equal 5?

b.   Write an equation that states that the contents of the parentheses must equal 5.
Then solve this equation. Did you get the same result as with David's method?

—————   *Further Guidance*   —————
section ends here.

10-48.   THE THREE METHODS

a.   Find the Math Notes box for this lesson and read it with your team.

b.   Match the names of approaches on the left with the examples on the right.

1. Rewriting

2. Looking inside

3. Undoing

i.   "If $3 + (4n - 4) = 12$, then $(4n - 4)$ must equal 9…"

ii.   "Subtracting is the opposite of adding, so for the equation $3(x - 7) + 4 = 23$, I can start by subtracting 4 from both sides…"

iii. "This problem might be easier if I turned $4(2x - 3)$ into $8x - 12$…"

10-49.   For each equation below, decide whether it would be best to rewrite, look inside, or undo. Then solve the equation, showing your work and writing down the name of the approach you used. Check your solutions, if possible.

a. $\frac{2x-8}{10} = 6$

b. $4 + (x \div 3) = 9$

c. $\sqrt{3x + 3} = 6$

d. $8 - (2x + 1) = 3$

e. $\sqrt{x} + 4 = 9$

f. $\frac{x}{3} - \frac{x}{9} = 6$

10-50.   Consider the equation $(x - 7)^2 = 9$.

a.   Solve this equation using *all three* approaches studied in this lesson. Make sure each team member solves the equation using all three approaches.

b.   Did you get the same solution using all three approaches? If not, why not?

c.   Of the three methods, which do you think was the most efficient method for this problem? Why?

METHODS AND MEANINGS

Methods to Solve One-Variable Equations

Here are three different approaches you can take to solve a one-variable equation:

Rewriting: Use algebraic techniques to rewrite the equation. This will often involve using the Distributive Property to get rid of parentheses. Then solve the equation using solution methods you know.

$$5(x-1) = 15$$
$$5x - 5 = 15$$
$$5x = 20$$
$$x = 4$$

Looking inside: Choose a part of the equation that includes the variable and is grouped together by parentheses or another symbol. (Make sure it includes *all* occurrences of the variable!) Ask yourself, "What must this part of the equation equal to make the equation true?" Use that information to write and solve a new, simpler equation.

$$5(x-1) = 15$$
$$5(\ 3 \ ) = 15$$
$$x - 1 = 3$$
$$x = 4$$

Undoing: Start by undoing the *last* operation that was done to the variable. This will give you a simpler equation, which you can solve either by undoing again or with some other approach.

$$\frac{5(x-1)}{5} = \frac{15}{5}$$
$$x - 1 = 3$$
$$+1 = +1$$
$$x = 4$$

Review & Preview

10-51. Read the statements made by Hank and Frank below.

Hank says, "The absolute value of 5 is 5."

Frank says, "The absolute value of –5 is 5."

a. Is Hank correct? Is Frank correct?

b. How many different values for x make the equation $|x| = 5$ true?

10-52. Use the results from problem 10-51 to help you find all possible values for x in each of the following equations.

    a. $|x| = 4$ b. $|x| = 100$

    c. $|x| = -3$ d. $|x - 2| = 5$

10-53. Which of the expressions below are equal to 1? (Note: More than one answer is possible!)

    a. $\frac{2x+3}{3+2x}$ b. $\frac{6x-12}{6(x-2)}$

    c. $\frac{(2x-3)(x+2)}{2x^2+x-6}$ d. $\frac{x}{2} \div \frac{2}{x}$

10-54. Solve the inequalities below. Write each solution as an inequality.

    a. $8 + 3x > 2$ b. $\frac{2}{3}x - 6 \le 2$

    c. $-2x - 1 < -3$ d. $\frac{5}{x} \le \frac{1}{3}$

10-55. For the equation $\frac{3}{200} + \frac{x}{50} = \frac{7}{100}$:

    a. Find a simpler equivalent equation (i.e., an equivalent equation with no fractions) and solve for x.

    b. Which method listed in this lesson's Math Notes box did you use in part (a)?

10-56. Mr. Nguyen has decided to divide $775 among his three daughters. If the oldest gets twice as much as the youngest, and the middle daughter gets $35 more than the youngest, how much does each child get? Write an equation and solve it. Be sure to identify your variables.

10.2.2 How many solutions?

Determining the Number of Solutions

$(x-1)^2 = 9$
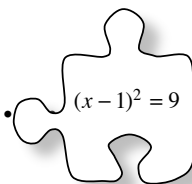
So far in this course you have seen many types of equations – some with no solution, some with one solution, others with two solutions, and still others with an infinite number of solutions! Is there any way to predict how many solutions an equation will have without solving it? Today you will focus on this question as you study quadratic equations written in perfect square form and equations with an absolute value. As you work with your team, ask the following questions:

Is there another way?

How do you see it?

Did you find all possible solutions?

10-57.   The quadratic equation below is written in **perfect square form**. It is called this because the term $(x-3)^2$ forms a square when built with tiles. Solve this quadratic equation using one of the methods you studied in Lesson 10.2.1.

$$(x-3)^2 = 12$$

a.   How many solutions did you find?

b.   Write your answer in **exact** form. That is, write it in a form that is precise and does not have any rounded decimals.

c.   Write your answer in **approximate** form. Round your answers to the nearest hundredth (0.01).

10-58.   THE NUMBER OF SOLUTIONS

The equation in problem 10-57 had two solutions. However, from your prior experience you know that some quadratic equations have no solutions and some have only one solution. How can you quickly determine how many solutions a quadratic equation has?

With your team, solve the equations below. Express your answers in both **exact form** and **approximate form**. Look for patterns among those with no solution and those with only one solution. Be ready to report your patterns to the class.

a. $(x+4)^2 = 20$ b. $(7x-5)^2 = -2$ c. $(2x-3)^2 = 49$

d. $(5-10x)^2 = 0$ e. $(x+2)^2 = -10$ f. $(x+11)^2 + 5 = 5$

10-59.   Use the patterns you found in problem 10-58 to determine quickly how many solutions each quadratic below has. You do not need to solve the equations.

a. $(5m-2)^2 + 6 = 0$ b. $(4+2n)^2 = 0$ c. $11 = (7+2x)^2$

10-60.   Consider the equation $|2x - 5| = 9$.

    a.   How many solutions do you think this equation has? Why?

    b.   Which of the three solution approaches do you think will work best for this equation?

    c.   With your team, solve $|2x - 5| = 9$. Record your work carefully as you go. Check your solution(s).

10-61.   The equation $|2x - 5| = 9$ from problem 10-60 had two solutions. Do you think all absolute-value equations must have two solutions? Consider this as you answer the questions below.

    a.   Can an absolute-value equation have no solution? With your team, create an absolute-value equation that has no solution. How can you be sure there is no solution?

    b.   Likewise, create an equation with an absolute value that will have only one solution. **Justify** why it will have only one solution.

10-62.   Is there a **connection** between how to determine the number of solutions of a quadratic in perfect square form and how to determine the number of solutions of an equation with an absolute value? In your Learning Log, describe this connection and explain how you can determine how many solutions both types of equations have. Be sure to include examples for each. Title this entry "Number of Solutions" and include today's date.
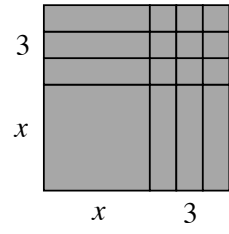

METHODS AND MEANINGS

MATH NOTES

Perfect Square Form of a Quadratic

When a quadratic equation is written in the form $(x - a)^2 = b^2$, such as the one below, we say it is in **perfect square form**. Notice that when the quadratic expression on the left side of the equation below is built with tiles, it forms a square, as shown at right.

$$(x + 3)^2 = 25$$

10-63.   Solve these equations, if possible. Each time, be sure you have found all possible
          solutions. Check your work and write down the name of the method(s) you used.

          a. $(x+4)^2 = 49$ b. $3\sqrt{x+2} = 12$

          c. $\frac{2}{x} + \frac{3}{10} = \frac{13}{10}$ d. $5(2x-1) - 2 = 13$

10-64.   Is $x = -4$ a solution to $\frac{1}{3}(2x+5) > -1$? Explain how you know.

10-65.   Multiply or divide the rational expressions below. Leave your answer in simplified
          form.

          a. $\frac{(x+4)(2x-1)(x-7)}{(x+8)(2x-1)(3x-4)} \div \frac{(4x-3)(x-7)}{(x+8)(3x-4)}$

          b. $\frac{2m^2+7m-15}{m^2-16} \cdot \frac{m^2-6m+8}{2m^2-7m+6}$

10-66.   An **exponent** is shorthand for repeated multiplication. For example, $x^3 = x \cdot x \cdot x$.
          Use the meaning of an exponent to rewrite each of the expressions below.

          a. $(3x-1)^2$ b. 7^4 c. m^3 d. w^{10}

10-67.   Factor each of the following expressions completely. Be sure to look for any
          common factors.

          a. $4x^2 - 12x$ b. $3y^2 + 6y + 3$

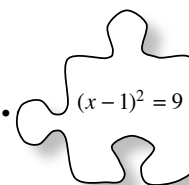   c. $2m^2 + 7m + 3$ d. $3x^2 + 4x - 4$

10-68.   Write and solve an equation to answer
          the question below. Remember to define
          any variables you use.

          Pierre's Ice Cream Shoppe charges
          $1.19 for a scoop of ice cream and $0.49
          for each topping. Gordon paid $4.55 for
          a three-scoop sundae. How many
          toppings did he get?

10.2.3 Which method is best?

....

More Solving and an Application

$(x-1)^2 = 9$

Recently you investigated three different approaches to solving one-variable equations: rewriting, looking inside, and undoing. Today you will use those approaches to solve new kinds of equations you have not solved before. You will also use your equation-writing skills to write an inequality for an application. As you work today, ask yourself these questions:

How can I represent it?

What is the best approach for this equation?

Have I found all of the solutions?

10-69. Solve these equations. Each time, be sure you have found all possible solutions. Check your work and write down the name of the method(s) you used.

a. $|x+1| = 5$

b. $(x-13)^3 = 8$

c. $2\sqrt{x-4} = 14$

d. $|4x+20| = 8$

e. $3(x+12)^2 = 27$

f. $6|x-8| = 18$

10-70. **RUB A DUB DUB**

Ernie is thinking of installing a new hot tub in his backyard. The company he will order it from makes square hot tubs, and the smallest tub he can order is 4 feet by 4 feet. He plans to add a 3-foot-wide deck on two adjacent sides, as shown in the diagram below. If Ernie's backyard (which is also a square) has 169 square feet of space, what are the possible dimensions that his hot tub can be? Write and solve an inequality that represents this situation. Be sure to define your variable.

3 ft

Hot Tub

3 ft **Deck**
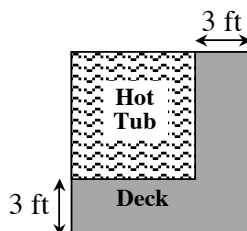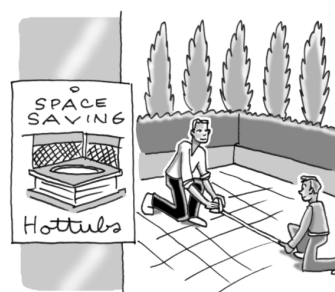
METHODS AND MEANINGS

Solving Absolute-Value Equations

To solve an equation with an absolute value algebraically, first determine the possible values of the quantity inside the absolute value.

For example, if $|2x+3|=7$, then the quantity $(2x+3)$ must equal 7 or –7.

With these two values, set up new equations and solve as shown below.

$$|2x+3|=7$$

$$2x+3=7 \quad \text{or} \quad 2x+3=-7$$
$$2x=4 \qquad\qquad 2x=-10$$
$$x=2 \qquad\qquad x=-5$$

Always check your solutions by substituting them into the original equation:

Test $x=2$: $\qquad |2(2)+3|=7$ ✓ True

Test $x=-5$: $\qquad |2(-5)+3|=7$ ✓ True

Review & Preview

10-71.   Sketch a graph of the inequality below. Shade the region containing the solutions of the inequality.

$$y > (x-4)(x+3)$$

10-72.   Jessie looked at the equation $(x-11)^2 = -4$ and stated, "This quadratic has no solutions!" How did she know?

10-73.   Solve these equations, if possible. Be sure to find all possible solutions. Check your work and write down the name of the method(s) you used.

a. $9(x-4)^2 = 81$

b. $|x-6| = 2$

c. $5 = 2 + \sqrt{3x}$

d. $2|x+1| = -4$

10-74.   Review what you know about solving inequalities by solving the inequalities below. Show your solutions on a number line.

     a. $6x - 1 < 11$ b. $\frac{1}{3}x \geq 2$

     c. $9(x - 2) > 18$ d. $5 - \frac{x}{4} \leq \frac{1}{2}$

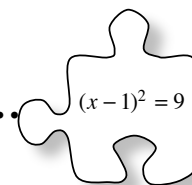10-75.   Multiply or divide the rational expressions below. Leave each answer in simplified form.

     a. $\frac{(x-3)^2}{2x-1} \cdot \frac{2x-1}{(3x-14)(x+6)} \cdot \frac{x+6}{x-3}$ b. $\frac{4x^2+5x-6}{3x^2+5x-2} \div \frac{4x^2+x-3}{6x^2-5x+1}$

10-76.   Use the meaning of an exponent to rewrite the expression $5x^3y^2$. Review the meaning of an exponent in problem 10-66 if necessary.

10.2.4   How can I solve the inequality?

$(x-1)^2 = 9$

Solving Inequalities with Absolute Value

The three approaches you have for solving equations can also be used to solve inequalities. While the one-variable inequalities you solve today look different than the ones in Chapter 9, the basic process for solving them is similar. As you solve equations and inequalities in today's lesson, ask yourself these questions:

<p style="text-align:center">How can I represent it?</p>

<p style="text-align:center">What connection can I make?</p>

10-77.   Solve the inequality $2x + 7 < 12$. Represent the solution on a number line.

     a.   What is the boundary point? Is it part of the solution? Why or why not?

     b.   In general, how do you find a boundary point? How do you find the solutions of an inequality after you have found the boundary point? Briefly review the process with your team.

10-78.   Now consider the inequality $|x-2| > 3$.

a.   Can you use the process from problem 10-77 to solve this inequality? How is it different from solving $|x-2| = 3$? Solve the inequality and represent your solution on a number line.

b.   How was solving $|x-2| > 3$ different from solving $2x+7 < 12$?

10-79.   Examine the graph of $y = |x-2|$ and $y = 3$ at right.
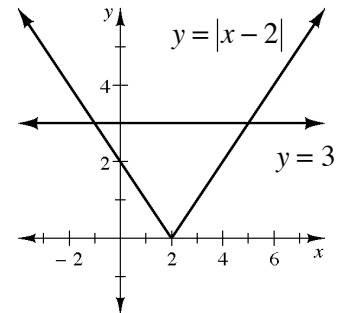
a.   How does this graph confirm your solution to $|x-2| > 3$ from problem 10-78? Be prepared to explain your thinking.

b.   How would the solution change for the inequality $|x-2| \le 3$? Draw this solution on a number line. Explain how the graph at right also confirms this solution.

c.   Now use the graph to predict the x-values that make $|x-2| \ge -1$ true. **Justify** your answer.

10-80.   Consider the quadratic inequality $x^2 + 2x + 1 < 4$.

a.   Solve for the boundary point(s). How many boundary points are there?

b.   Place the boundary point(s) on a number line. How many regions do you need to test?

c.   Test each region and determine which one(s) make the inequality true. Identify the solution region(s) on the number line.

d.   Confirm your solution by graphing $y = x^2 + 2x + 1$ and $y = 4$ on the same set of axes on graph paper. Highlight the portion of the parabola that lies below the line $y = 4$. Does this confirm your solution to part (c)?

10-81.   Revisit the graph from problem 10-79. Use it to write an inequality involving $|x-2|$ that has no solution.

10-82.   In your Learning Log, explain how you can solve an inequality that has an absolute value. You may include an explanation of the graphical process if you choose. Then make up your own example problem and show how that problem is solved. Title this entry "Solving Inequalities with Absolute Value" and include today's date.

10-83. Examine the rectangle formed by the tiles shown at right. Write the area of the rectangle as a product and as a sum.

10-84. How many solutions does each quadratic equation below have?

a. $6x^2 + 7x - 20 = 0$

b. $m^2 - 8m + 16 = 0$

c. $2r^2 + r + 3 = 0$

d. $(2k + 1)^2 = 0$

10-85. Find the equation of the line perpendicular to $y = -\frac{2}{3}x - 7$ that goes through the point (–6, 9).

10-86. On graph paper, graph the system of inequalities below. Carefully shade the region that represents the solution to both inequalities.

$$y \le -|x - 2| + 3$$
$$y \ge -1$$

10-87. Multiply or divide the expressions below. Leave your answer as simplified as possible.

a. $\frac{8x^2 - 12x - 8}{2x^2 - 5x - 3} \cdot \frac{x^2 + 2x - 15}{6x - 12}$

b. $\frac{7x^2 + 5x - 2}{x^2 + 2x - 8} \div \frac{3x^2 - 2x - 5}{3x^2 - 11x + 10}$

10-88. Solve the equations and inequalities below. Check your solution(s), if possible.
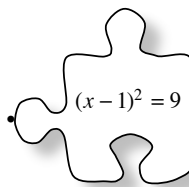
a. $300x - 1500 = 2400$

b. $\frac{3}{2}x = \frac{5}{6}x + 2$

c. $x^2 - 25 \le 0$

d. $|3x - 2| > 4$

10.2.5 How can I solve this inequality?

$(x-1)^2 = 9$

Solving Absolute-Value and Quadratic Inequalities

Today you will finish your focus on solving equations and inequalities. By the end of the lesson today, you will have the tools to solve complex equations and inequalities.

10-89. At right is a graph showing $y = x^2 + x - 6$ and $y = 3x + 2$. Use the graph to find the solutions for $x^2 + x - 6 \geq 3x + 2$. Indicate your answer on a number line.
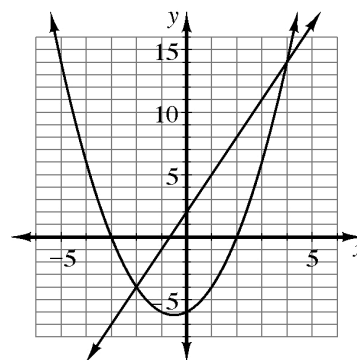
10-90. Solve the inequalities below, if possible, and represent your solution on a number line.

a. $|x + 2| > 1$ b. $x^2 + x - 12 < 0$

c. $|2(x - 1)| \geq 0$ d. $9x - 4 \leq 6 - x$

e. $|3x - 11| < -2$ f. $(x - 2)^2 > 7$

10-91. FOG CITY

San Francisco is well known for its fog: very thick, low-lying clouds that hide its hills. One foggy day, Penelope was practicing kicking a football on the football field of her school. Once she kicked the football so high that it disappeared into the fog! If the height h of the ball (in feet) could be represented at time t (in seconds) by the equation $h = -16t^2 + 96t$, and if the fog was 140 feet off the ground, during what times of its flight was the ball not visible? Explain how you got your answer.

10-92.   **PULLING IT TOGETHER**

Now that you have the skills necessary to solve
many interesting equations and inequalities,
work with your team to solve the equation
below. (This equation was first introduced in
Lesson 10.2.1.) Show your solutions on a
number line and be prepared to share your
solving process with the class.

$$(\sqrt{|x+5|} - 6)^2 + 4 = 20$$

Review & Preview

10-93.   Review the meaning of an exponent in problem 10-66. Then use its meaning to
rewrite the expression $(y-2)^3$.

10-94.   How many solutions does the equation $|7 - 3x| + 1 = 0$ have? Explain how you
know.

10-95.   Solve the equations and inequalities below, if possible.

a. $\sqrt{x-1} + 13 = 13$ b. $6|x| > 18$ c. $|3x - 2| \le 2$

d. $\frac{4}{5} - \frac{2x}{3} = \frac{3}{10}$ e. $(4x - 2)^2 \le 100$ f. $(x-1)^3 = 8$

10-96.   On graph paper, graph a line with slope $-\frac{2}{3}$ that goes through the point (4, –3).

a.   Find the equation of the line.

b.   Is this line perpendicular to the line $6x - 4y = 8$? Explain how you know.

10-97.   Simplify the rational expressions below as much as possible.
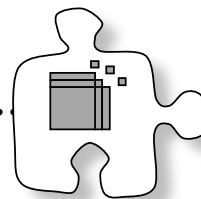
a. $\frac{(x-4)^3(2x-1)}{(2x-1)(x-4)^2}$ b. $\frac{7m^2 - 22m + 3}{3m^2 - 7m - 6}$

c. $\frac{(z+2)^9(4z-1)^7}{(z+2)^{10}(4z-1)^5}$ d. $\frac{(x+2)(x^2 - 6x + 9)}{(x-3)(x^2 - 4)}$

10-98.   **Multiple Choice:** Which of the graphs below shows the solutions for $y < -\frac{2}{5}x + 1$?

a.

b.

c.

d.
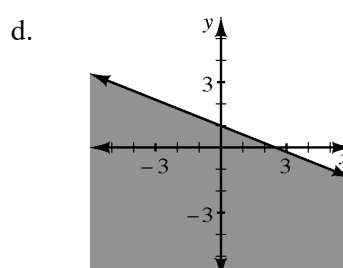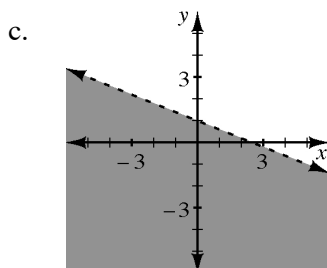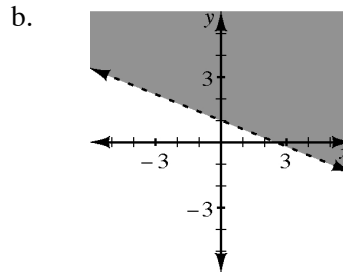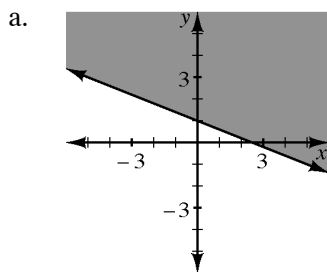
10.3.1   How can I make it a perfect square?

Completing the Square

You have learned many ways to solve quadratic equations so far in this course. Sometimes, using the Quadratic Formula can be complicated and messy, while solving equations in perfect square form (such as $(x+2)^2 = 3$) can be very straightforward. Therefore, it is sometimes convenient to change a quadratic equation from standard form into perfect square form. One method that you will investigate in this lesson is called **completing the square**.

10-99.   Review what you know about solving quadratic equations as you solve the two equations below. Be ready to share your method(s) with the class.
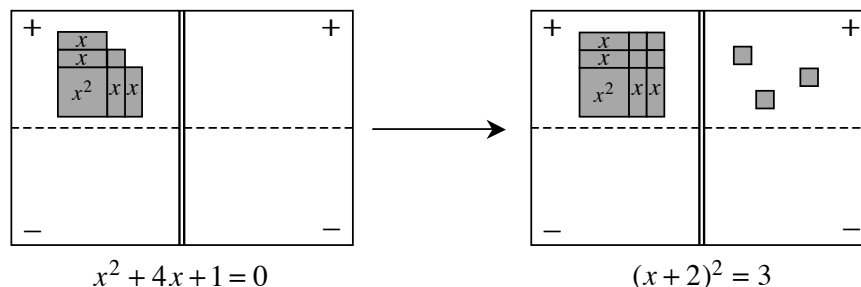
a. $x^2 + 4x + 1 = 0$

b. $(x + 2)^2 = 3$

10-100.   With your team, and then with the class, discuss the following questions.

a.   Examine the solutions to $x^2 + 4x + 1 = 0$ and $(x + 2)^2 = 3$. What do you notice? What does this tell you about the two equations? Verify your conclusion algebraically.

b.   Of the methods used in problem 10-99, which do you think was most efficient and straightforward?
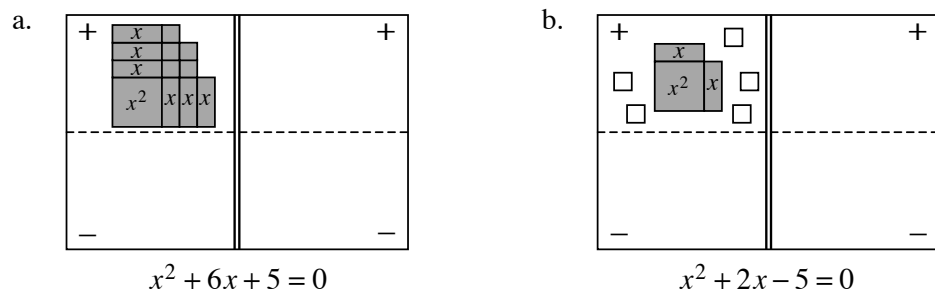
10-101. COMPLETING THE SQUARE

With your team, examine how the two different equations from problem 10-99 can be represented using tiles on an equation mat, shown below. Then answer questions (a) and (b) below.

$\blacksquare = +1$
$\square = -1$

$$x^2 + 4x + 1 = 0 \qquad (x+2)^2 = 3$$

a. What "legal" move can be done to the equation $x^2 + 4x + 1 = 0$ that will result in the equation $(x+2)^2 = 3$?

b. Changing a quadratic equation into perfect square form is also known as "completing the square." Why is this name appropriate?

10-102. Use the process from problem 10-101 to change the quadratics below into perfect square form. Then solve the resulting quadratics. Building the squares with algebra tiles may be useful. Record your work on the resource page provided by your teacher.

$\blacksquare = +1$
$\square = -1$

a.

$$x^2 + 6x + 5 = 0$$

b.

$$x^2 + 2x - 5 = 0$$

10-103. The problems below introduce different situations that can arise while completing the square. Carefully choose what to add to both sides of each equation below to change the quadratics into perfect square form. Then solve the resulting quadratic. Again, building the equations with algebra tiles may be useful. Record your work on the resource page provided by your teacher.

■ = +1
□ = −1

a.

$x^2 - 6x - 1 = 0$

b.

$x^2 + 4x - 5 = 0$

c.
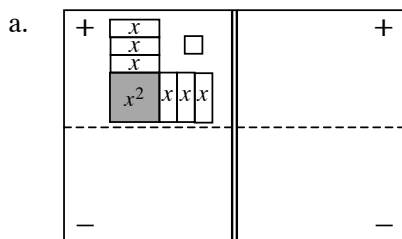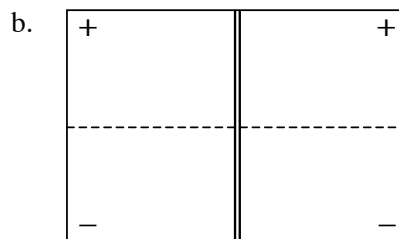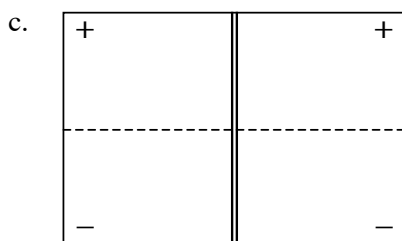
$x^2 - 2x - 3 = 1$

10-104. Use algebra tiles to change $4x^2 + 12x + 3 = 10$ into perfect square form. Then solve the resulting quadratic equation.

METHODS AND MEANINGS

Forms of a Quadratic Equation

There are three main forms of a quadratic equation: standard form, factored form, and perfect square form. Study the examples below. Assume that $a \neq 0$ and that the meaning of a, b, and c are different for each form below.

Standard form: Any quadratic written in the form $ax^2 + bx + c = 0$.

Factored form: Any quadratic written in the form $a(x+b)(x+c) = 0$.

Perfect square form: Any quadratic written in the form $a(x+b)^2 = c$.

10-105.   Use your understanding of the number 1 to simplify the rational expressions below.

a. $\dfrac{(x-3)(2x+9)(4x-3)}{(2x+9)(5x+1)(x-3)}$

b. $\dfrac{25x^2+20x+4}{25x^2-4}$

c. $\dfrac{16x^2+24x+8}{2x^2-2x-4}$

d. $\dfrac{24xy^2}{36x^2y}$

10-106.   Solve the quadratic equation below *twice*: once using the Quadratic Formula and once by completing the square and solving the quadratic in perfect square form. You should get the same result using both methods. What happened?

$$x^2 + 6x + 11 = 0$$

10-107.   Solve the inequalities and equations below, if possible. Represent your solution on a number line.

a. $|x| + 3 < 5$

b. $5(2x+1) \geq 30$

c. $\dfrac{1}{x} - \dfrac{5}{2} = \dfrac{3}{2}$

d. $-5 - x > 3 - x$

e. $3\sqrt{4-x} + 1 = 13$

f. $|x+1| \leq 4$

10-108.   Verify your solution to part (f) of problem 10-107 by graphing the functions below on the same set of axes. Highlight the portion(s) of the graph for which $|x+1| \leq 4$.

$$y = |x+1|$$
$$y = 4$$

10-109.   Write and solve an equation to solve the problem below. State your solution as a sentence.

Shu Min currently has 18 CDs, and her music club sends her three more CDs each month. Her brother, Wei, currently has 22 CDs and buys two more CDs each month with his allowance. Their CD holder can only hold 80 CDs. After how many months will their CD holder be full?

10.3.2 How can I generalize?

More Completing the Square
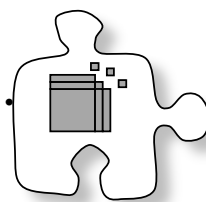
Today you will learn more about completing the square and will **generalize** how to complete the square for any quadratic in standard form.

10-110. Determine the number of solutions for each quadratic equation below by first completing the square (using algebra tiles or drawing a diagram). Then explain how you can quickly determine how many solutions a quadratic equation has once it is written in perfect square form.

a. $x^2 - 6x + 7 = 0$ b. $m^2 + 12m + 37 = 0$

c. $p^2 + 2p + 1 = 0$ d. $k^2 - 4k + 9 = 0$

10-111. Examine the results of your work in problem 10-110 and look for ways to **generalize** the process of completing the square. In other words, how can you change a quadratic into perfect square form without using tiles or drawing a diagram? It may help to make a table like the one started below. Then answer the questions that follow.

Standard Form	Perfect Square Form
$x^2 - 6x + 7 = 0$	

a. Describe any patterns you found when comparing a quadratic written in standard form with its corresponding equation in perfect square form.

b. When a quadratic is changed to perfect square form, how can you predict what will be in the parentheses? For example, if you want to change $x^2 + 10x - 3 = 0$ into perfect square form, what will be the dimensions of the square?

c. To complete the square, you often need to add some unit tiles to both sides of the equation. How can you predict how many tiles will need to be added or removed?

10-112. Use your generalized process of completing the square to rewrite and solve each quadratic equation below.

a. $w^2 + 28w + 52 = 0$ b. $x^2 + 5x + 4 = 0$

c. $k^2 - 16k - 17 = 0$ d. $z^2 - 1000z + 60775 = 0$

10-113.   What is the slope of the line passing through the points (4, –8) and (–3, 12)?

10-114.   Use your generalized process of completing the square to rewrite and solve each quadratic equation below.

  a. $x^2 + 4x = -3$ b. $x^2 - 8x + 7 = 0$ c. $x^2 - 24x + 129 = 0$

10-115.   Multiply or divide the rational expressions below. Leave your answers in simplified form.

  a. $\frac{4x^2+x-14}{3x^2-11x+6} \div \frac{4x-7}{x-3}$ b. $\frac{5x^2-8x-4}{x^2-9x-22} \cdot \frac{x^2-4}{5x^2+22x+8}$

10-116.   Solve the following equations and inequalities, if possible. Represent each solution on a number line.

  a. $\frac{3}{9} - \frac{x}{3} = \frac{x}{5}$ b. $(3+x)^2 < 9$ c. $8|x+1| \geq 64$

  d. $11 - \sqrt{x+3} = 13$ e. $\frac{x}{8} = \frac{2}{x}$ f. $|x-5| + 1 > 0$

10-117.   On graph paper, graph the inequality $y \leq |x| + 2$.

10-118.   Aura currently pays $800 each month to rent her apartment. Due to inflation, however, her rent is increasing by $50 each year. Meanwhile, her monthly take-home pay is $1500 and she predicts that her monthly pay will only increase by $15 each year. Assuming that her rent and take-home pay will continue to grow linearly, will her rent ever equal her take-home pay? If so, when? And how much will rent be that year?

10.4.1   How can I rewrite it?

$x^3 = x \cdot x \cdot x$

Simplifying Exponential Expressions
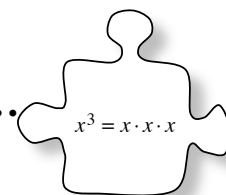
In Section 10.1, you used the property of the number 1 to simplify rational expressions. Today you will examine how to simplify expressions with exponents. Using patterns, you will develop strategies to simplify expressions when the exponents are too large to expand on paper.

10-119.   You have seen that you can rewrite expressions using the number 1. You can also simplify using the meaning of an exponent.

An **exponent** is shorthand for repeated multiplication. For example, $n^4 = n \cdot n \cdot n \cdot n$.

a.   Expand each of the expressions below. For example, to expand x^3, you would write: $x \cdot x \cdot x$.

   i. y^7 ii. $5(2m)^3$ iii. $(x^3)^2$ iv. $4x^5y^2$

b.   Simplify each of the expressions below using what you know about exponents and the number 1. Start by expanding the exponents, and then simplify your results.

   i. $\frac{x \cdot x \cdot x}{x}$ ii. $\frac{x^5}{x^2}$ iii. $x^2 \cdot x^3$ iv. $k^3 \cdot k^5$

   v. $\frac{16k^3}{8k^2}$ vi. $m^6 \cdot m$ vii. $x^4 \cdot x^5 \cdot x^3$ viii. $\frac{6x^3y}{2y}$

   challenge: $\frac{5x^{50}}{10x^{15}}$

10-120.   Simplify each of the expressions below. Start by expanding the exponents, and then simplify your results. Look for patterns or possible shortcuts that will help you simplify more quickly. Be prepared to **justify** your patterns or shortcuts to the class.

a. $y^5 \cdot y^2$

b. $\frac{w^5}{w^2}$

c. $(x^2)^4$

d. $x^{10} \cdot x^{12}$

e. $\frac{13p^4q^5}{p^2q^2}$

f. $\left(\frac{x^2}{y}\right)^3$

g. $5h \cdot 2h^{24}$

h. $\frac{10m^{30}}{2m^8}$

i. $(3k^{20})^4$

j. $\frac{24hg^2}{3hg^9}$

k. $\left(\frac{m^3}{n^{10}}\right)^4$

l. $w^4 \cdot p \cdot w^3$

10-121.   Work with your team to write four exponent problems, each having a simplification of x^{12}. At least one problem must involve multiplication, one must involve grouping, and one must involve division. Be creative!

10-122.   Gerardo is simplifying expressions with very large exponents. He arrives at each of the results below. For each result, decide if he is correct and **justify** your answer using the meaning of exponents.

a. $\frac{x^{150}}{x^{50}} \Rightarrow x^3$

b. $y^{20} \cdot y^{41} \Rightarrow y^{61}$

c. $(2m^2n^{15})^3 \Rightarrow 2m^6n^{45}$

METHODS AND MEANINGS

Completing the Square

Previously in this course, you have learned to solve quadratic equations by graphing, factoring, and using the Quadratic Formula. Another way to solve a quadratic equation is by **completing the square**. See the example below.

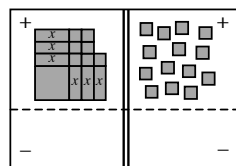**Example:** Solve for x by completing the square: $x^2 + 6x + 7 = 14$

First, use algebra tiles or a generic rectangle to determine if $x^2 + 6x + 7$ is already a perfect square.

$$x^2 + 6x + 7 = 14$$

Using algebra tiles (shown at right), you can see that there are not enough tiles to build a complete square. Therefore, two unit tiles must be added to both sides of the equation to complete the square.

Add 2 unit tiles to complete the square.

Notice that the square has side length $x + 3$. Any quadratic of the form $x^2 + bx + c$ will be converted to a square of side length $x + \frac{b}{2}$.
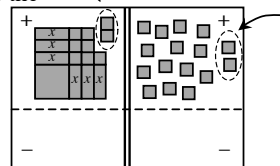
$$x^2 + 6x + 9 = 16$$

Now rewrite the expressions on each side of the equation so that the equation is in perfect square form. Then solve this equation by undoing the square and subtracting 3 from both sides.

$$(x + 3)^2 = 16$$
$$x + 3 = \pm 4$$
$$x = -3 \pm 4$$
$$x = 1 \text{ or } -7$$

As always, be sure to check your solutions in the original equation.

$$(1)^2 + 6(1) + 7 = 14$$
$$(-7)^2 + 6(-7) + 7 = 14$$

10-123. Use what you have learned about exponents to rewrite each of the expressions below.

   a. $\dfrac{h^9}{h^{11}}$

   b. $x^3 \cdot x^4$

   c. $(3k^5)^2$

   d. $n^7 \cdot n$

   e. $\dfrac{16x^4y^3}{2x^4}$

   f. $4xy^3 \cdot 7x^2y^3$

10-124. Lacey and Haley are simplifying expressions.

   a. Haley simplified $x^3 \cdot x^2$ and gets x^5. Lacey simplified $x^3 + x^2$ and got the same result! However, their teacher told them that only one simplification is correct. Who simplified correctly and how do you know?

   b. Haley simplifies $3^5 \cdot 4^5$ and gets the result 12^{10}, but Lacey is not sure. Is Haley correct? Be sure to **justify** your answer.

10-125. On your paper, draw the algebra tiles to represent the equation $x^2 + 2x = 8$ on an equation mat.

   a. How many tiles do you need to add or remove from each side of the equation to complete the square?

   b. Write the equation in perfect square form.

10-126. Find a rule that represents the number of tiles in the tile pattern at right.

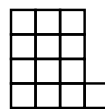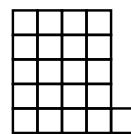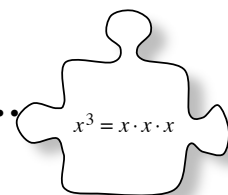Figure 0   Figure 1   Figure 2

10-127. Solve the equations and inequalities below. If necessary, write your solutions in approximate form.

   a. $900x - 200 = 500x + 600$

   b. $3k^2 - 15k + 14 = 0$

   c. $|x - 4| < 6$

   d. $\dfrac{7}{3} + \dfrac{x}{2} = \dfrac{6x-1}{6}$

10.4.2 How can I rewrite it?

Zero and Negative Exponents

$x^3 = x \cdot x \cdot x$

In Lesson 10.4.1, you used the meaning of an exponent to rewrite expressions such as $y^4 \cdot y^2$ and $(x^2 y)^3$. Today you will use the patterns you discovered to learn how to interpret expressions with exponents that are negative or zero.

10-128. Review what you learned about exponents in Lesson 10.4.1 to rewrite each expression below as simply as possible. If you see a pattern or know of a shortcut, be sure to share it with your teammates.

    a. $x^7 \cdot x^4$
    b. $(x^3)^3$
    c. $\frac{m^{14}}{m^2}$

    d. $(x^2 y^2)^4$
    e. $\frac{x^2 y^{11}}{x^5 y^3}$
    f. $\frac{2x^{12}}{8x^2}$

10-129. With your study team, summarize the patterns you found in problem 10-128. For each one, simplify the given expression and write an expression that represents its **generalization**. Then, in your own words, explain why the pattern works.

	Expression	Generalization	Why is this true?
a.	$x^{25} \cdot x^{40} = ?$	$x^m \cdot x^n = ?$	
b.	$\frac{x^{36}}{x^{13}} = ?$	$\frac{x^m}{x^n} = ?$	
c.	$(x^5)^{12} = ?$	$(x^m)^n = ?$	

10-130. Describe everything you know about $\frac{x^m}{x^m}$. What is its value? How can you rewrite it using a single exponent? What new conclusions can you draw? Be prepared to explain your findings to the class.

10-131.   Problem 10-130 helped you recognize that
$x^0 = 1$. Now you will similarly use division to
explore the meaning of x^{-1}, x^{-2}, etc. Simplify
each of the expressions below *twice*:

- Once by expanding the terms and
  simplifying.

- Again by using your new pattern for
  division with exponents.

Be ready to discuss the meaning of negative
exponents with the class.

a. $\dfrac{x^4}{x^5}$ b. $\dfrac{x^2}{x^4}$ c. $\dfrac{x^7}{x^{10}}$

10-132.   Use your exponent patterns to rewrite each of the expressions below. For example,
if the original expression has a negative exponent, then rewrite the expression so that
it has no negative exponents – and vice versa. Also, if the expression contains
multiplication or division, then use your exponent rules to simplify the expression.

a. k^{-5} b. m^0 c. $x^{-2} \cdot x^5$ d. $\dfrac{1}{p^2}$

e. $\dfrac{y^{-2}}{y^{-3}}$ f. $(x^{-2})^3$ g. $(a^2 b)^{-1}$ h. $\dfrac{1}{x^{-1}}$

10-133.   EXPONENT CONCENTRATION

Split your team into two pairs and decide which is Team A and which is Team B.
Your teacher will distribute a set of cards for a game described below.

- Arrange the cards face down in a rectangular grid.

- Team A selects and turns over two cards.

- If Team A thinks the values on the cards are equivalent, they must **justify**
  this claim to Team B. If everyone in Team B agrees, Team A takes the pair.
  If the values are not equivalent, Team A returns both cards to their original
  position (face down). This is the end of the turn for Team A.

- Team B repeats the process.

- Teams alternate until no cards remain face down. The team with the most
  matches wins.

10-134. In your Learning Log, describe the meaning of zero and negative exponents. That is, explain how to interpret x^0 and x^{-1}. Title this entry "Zero and Negative Exponents" and include today's date.

LOOKING DEEPER

Inductive and Deductive Reasoning

When you make a conclusion based on a pattern, you are using **inductive reasoning**. So far in this course, you have used inductive reasoning repeatedly to **generalize** patterns. For example, in Lesson 10.4.1 and in this lesson, you used patterns to generalize the facts that $x^m x^n = x^{m+n}$ and $(x^m)^n = x^{mn}$.

However, you can also make a conclusion based on facts, using logic. This is called **deductive reasoning**. You used deductive reasoning during this lesson when you determined that $x^{-1} = \frac{1}{x}$. See the logical deduction below.

Statement	Reason
Since $\frac{x^4}{x^5} = x^{-1}$,	This is true because $\frac{x^m}{x^n} = x^{m-n}$.
And since $\frac{x^4}{x^5} = \frac{x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} = \frac{1}{x}$,	This is true because $\frac{x}{x} = 1$.
Therefore, $x^{-1} = \frac{1}{x}$.	$\frac{x^4}{x^5}$ equals both x^{-1} and $\frac{1}{x}$, so $x^{-1} = \frac{1}{x}$. (This is called the Transitive Property of Equality.)

Review & Preview

10-135. Which of the expressions below are equivalent to $16x^8$? Make sure you find *all* the correct answers!

a. $(16x^4)^2$

b. $8x^2 \cdot 2x^6$

c. $(2x^2)^4$

d. $(4x^4)^2$

e. $(2x^4)^4$

f. $(\frac{1}{16}x^{-8})^{-1}$

10-136. Write the inequality represented by the graph at right.
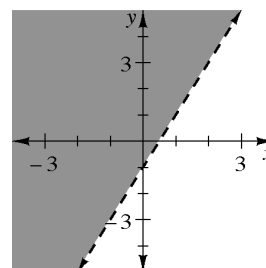
10-137. Solve the system of equations below using any method. Check your solution.

$$8y - 1 = x$$
$$10y - x = 5$$

10-138. Chad is entering a rocket competition. He needs to program his rocket so that when it is launched from the ground, it lands 20 feet away. In order to qualify, it must be 100 feet off the ground at its highest point. What equation should he program into his rocket launcher to win? Let x represent the distance from the launch pad in feet and y represent the height of the rocket in feet. Draw a sketch of the rocket's path.

10-139. Solve the quadratic equation below *twice*, once using the Quadratic Formula and once by completing the square. Which was easier?

$$x^2 - 10x + 21 = -4$$
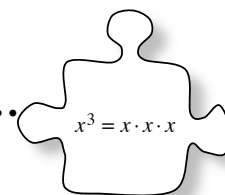
10-140. Simplify the rational expressions below.

a. $\dfrac{x^2 - 8x + 16}{3x^2 - 10x - 8}$

b. $\dfrac{10x + 25}{2x^2 - x - 15}$

c. $\dfrac{9x^4 y^3 z}{3x^4 y^3 z}$

10.4.3 How can I rewrite it?

Fractional Exponents and Scientific Notation

$x^3 = x \cdot x \cdot x$

So far you have discovered ways to deal with exponents when multiplying and dividing. You have also found ways to interpret expressions when the exponent is zero or negative. But what if the exponent is a fraction? And how can exponents help you rewrite numbers?

Today you will develop an understanding for fractional exponents and learn about scientific notation, a way to use exponents to rewrite very large or very small numbers.

10-141. FRACTIONAL EXPONENTS

What happens when an exponent is a fraction? Consider this as you answer the questions below.

a. Calculate $9^{1/2}$ with your scientific calculator. What is the result? Also use your calculator to find $49^{1/2}$ and $100^{1/2}$. What effect does having $\frac{1}{2}$ in the exponent appear to have?

b. Based on your observation in part (a), predict the value of $4^{1/2}$ and $(7^{1/2})^2$. Then confirm your prediction with your calculator.

c. Was the reasoning you used in part (a) an example of inductive or deductive reasoning? Refer to the Lesson 10.4.2 Math Notes box to help you decide.

10-142. Danielle wants to understand why $9^{1/2}$ is the same as $\sqrt{9}$. Since exponents represent repeated multiplication, Danielle decided to rewrite the number 9 as $3 \cdot 3$. She then reasoned that $9^{1/2}$ is asking for 1 of the 2 repeated factors with a product of 9.

a. Using Danielle's logic, find $16^{1/2}$. Confirm your answer with your calculator.

b. What is the value of $8^{1/3}$? $125^{1/3}$? How can you use the same reasoning to find these values? Confirm your answers with your calculator.

c. What about $27^{2/3}$? $32^{3/5}$? $25^{3/2}$? Use your calculator to find each of these values. Then apply Danielle's logic to make sense of what each of these expressions mean. Share any insight with your team members.

d. Another name for $x^{1/3}$ is "cube root." This can be written $\sqrt[3]{x}$. What would be the notation for $x^{1/5}$? What should it be called?

10-143.   Now that you have many tools to rewrite expressions with exponents, use these tools together to rewrite each of the expressions below. For example, $\sqrt{2^5} = (2^5)^{1/2} = 2^{5/2}$, since taking the square root of a number is the same as raising that number to the one-half power.

a. $(\sqrt{3})^4$ b. $9^{7/2}$ c. $\sqrt[3]{2^5}$

10-144.   Match each expression below on the left (letters (a) through (h)) with an equivalent expression on the right (numbers 1 through 8). Assume $x > 0$.

a. $\sqrt{x^3}$ e. $\sqrt[3]{x^2}$ 1. x^{-2} 5. $\sqrt{x}$

b. $\dfrac{x^2}{x^5}$ f. 1 2. x 6. $x^{2/3}$

c. $(\sqrt[3]{x})^5$ g. $x^{-3}x^4$ 3. $x^{3/2}$ 7. $x^{5/3}$

d. $\dfrac{1}{x^2}$ h. $(x^{1/4})^2$ 4. x^0 8. x^{-3}

10-145.   Exponents can also help you represent very large (and very small) numbers. For example, a very large number like the one below can be difficult to write out in complete form (called **standard form**).

3,000,000,000,000,000,000,000,000,000,000
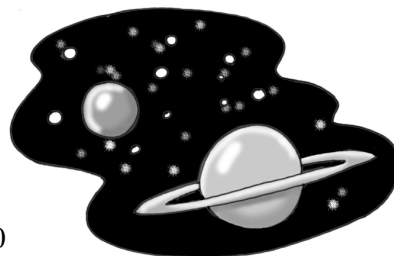
Instead, you can write this number using **scientific notation**: $3 \cdot 10^{30}$. This shorthand notation is not only easier to write, but it also gives you an immediate sense of how large the number is. Since 10^{30} is 10 multiplied by itself thirty times, then you know that $3 \cdot 10^{30}$ is the number 3 with 30 zeros after it.
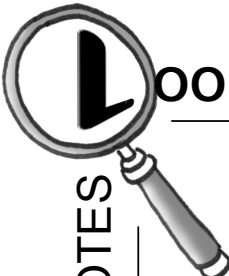
Similarly, $1.4 \cdot 10^8$ is 1.4 multiplied by 10 eight times. Thus $1.4 \cdot 10^8 = 140,000,000$.

Scientific notation is also useful for writing small numbers, such as 0.00024. Since $0.00024 = 24 \cdot \frac{1}{10,000}$, you can rewrite the number using scientific notation: $2.4 \cdot 10^{-4}$.

a.   Scientists claim that the earth is about $4.6 \cdot 10^9$ years old. Write this number in standard form.

b.   The average distance between the Earth and the sun is about 150,000,000,000 meters. Translate this number into scientific notation.

c.   It takes light about $3.3 \cdot 10^{-9}$ seconds to travel one meter. Express this number in standard form.

10-146. Scientific notation is not only a convenient way to write very large and very small numbers, but it also makes them easier to put into your calculator.

a. For example, multiply $5000 \cdot 20{,}000{,}000{,}000{,}000$ and write the answer in standard form. If these numbers cannot be entered into your calculator, then multiply them by hand on your paper.

b. Now multiply these same numbers by first changing each into scientific notation. For example, $5000 = 5 \cdot 10^3$. Express your answer in scientific notation.

c. Which method was easier and why?

MATH NOTES

LOOKING DEEPER

Laws of Exponents

In the expression x^3, x is the **base** and 3 is the **exponent**.

$$x^3 = x \cdot x \cdot x$$

The patterns that you have been using during this section of the book are called the **laws of exponents**. Here are the basic rules with examples:

| Law	Examples	
$x^m x^n = x^{m+n}$ for all x	$x^3 x^4 = x^{3+4} = x^7$	$2^5 \cdot 2^{-1} = 2^4$
$\frac{x^m}{x^n} = x^{m-n}$ for $x \neq 0$	$x^{10} \div x^4 = x^{10-4} = x^6$	$\frac{5^4}{5^7} = 5^{-3}$
$(x^m)^n = x^{mn}$ for all x	$(x^4)^3 = x^{4 \cdot 3} = x^{12}$	$(10^5)^6 = 10^{30}$
$x^0 = 1$ for $x \neq 0$	$\frac{y^2}{y^2} = y^0 = 1$	$9^0 = 1$
$x^{-1} = \frac{1}{x}$ for $x \neq 0$	$\frac{1}{x^2} = (\frac{1}{x})^2 = (x^{-1})^2 = x^{-2}$	$3^{-1} = \frac{1}{3}$
$x^{m/n} = \sqrt[n]{x^m}$ for $x \geq 0$	$\sqrt{k} = k^{1/2}$	$y^{2/3} = \sqrt[3]{y^2}$

Review & Preview

10-147. Simplify each of the expressions below. Your final simplification should contain no negative exponents.

a. $(5x^3)(-3x^{-2})$

b. $(4p^2 q)^3$

c. $\frac{3m^7}{m^{-1}}$

10-148.   Neil A. Armstrong was the first person ever to walk on the moon. After his historic landing on July 20, 1969, he stepped onto the moon's surface and spoke the famous phrase, "That's one small step for a man, one giant leap for mankind."
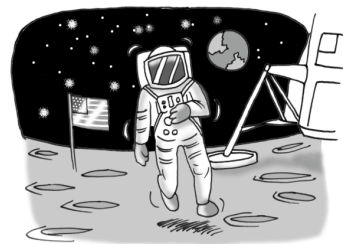
His craft, Apollo 11, traveled 238,900 miles from Earth to reach the moon. How many feet was this? Express your answer in both standard form and in scientific notation. Round your decimal to the nearest hundredth. (Note: There are 5280 feet in each mile.)

10-149.   Solve the system of equations below by graphing. Write your solution(s) in (x, y) form.

$$y = -4x - 2$$
$$y = x^2 - 3x - 4$$

10-150.   Write and solve an inequality to represent the situation below. Write your solution as a sentence.

Vinita wants to rent a skateboard and only has $20. She found out that the shop will charge her $8 to rent the skateboard plus $3.75 per hour. She does not know how long she wants to rent it. What are her options?

10-151.   A motion detector can record the distance between a moving person and the detector. Examine the graphs below, each of which was generated when a different person walked in front of a motion detector. For each graph, describe the motion of the person. Did they walk quickly? Did they walk slowly? In what direction did they walk? If the motion is not possible, explain why not.

a.

Distance (ft)

Time (seconds)

b.

Distance (ft)

Time (seconds)

c.

Distance (ft)

Time (seconds)
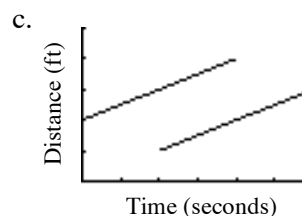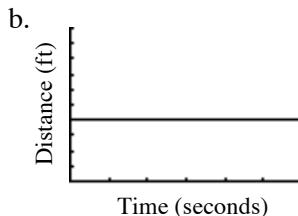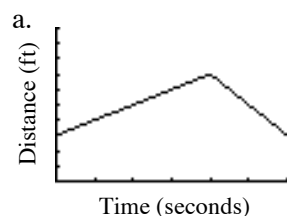
Chapter 10 Closure   What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for **connections** between ideas as well as **connections** with material you learned previously.

①   TEAM BRAINSTORM

With your team, brainstorm a list for each of the following topics. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Topics:	What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.
Ways of Thinking:	What Ways of Thinking did you use in this chapter? When did you use them?
Connections:	What topics, ideas, and words that you learned *before* this chapter are **connected** to the new ideas in this chapter? Again, make your list as long as you can.

②   MAKING CONNECTIONS

The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

absolute value	**base**	boundary point
completing the square	equivalent equations	**exponent**
fraction buster	inequality	**looking inside**
number line	**perfect square form**	quadratic equation
Quadratic Formula	**rational expression**	**rewriting**
scientific notation	simplifying	solution
standard form for quadratics	**undoing**	

Continues on next page →

② *Continues from previous page.*

Make a concept map showing all of the **connections** you can find among the key words and ideas listed on the previous page. To show a **connection** between two words, draw a line between them and explain the **connection**, as shown in the example below. A word can be **connected** to any other word as long as there is a **justified connection**. For each key word or idea, provide a sketch that illustrates the idea (see the example below).
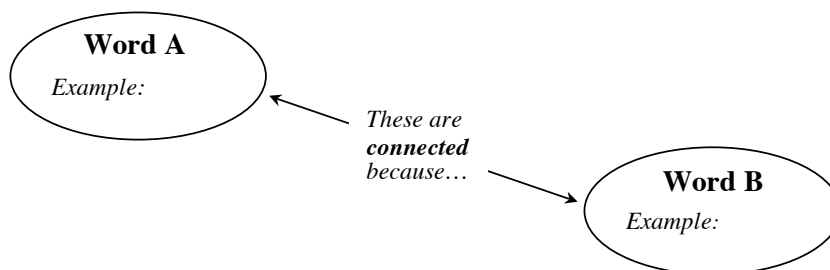
Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the **connections** explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will give you directions for exactly how to do this.

④ WHAT HAVE I LEARNED?

This section will help you evaluate which types of problems you have seen with which you feel comfortable and those with which you need more help. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

CL 10-152. Simplify the following expressions.

a. $\dfrac{x^2 y^3}{xy^2}$

b. $\dfrac{(x+2)^2(x-4)}{(x+2)(x-4)}$

c. $\dfrac{x^2-x-6}{x-3}$

d. $\dfrac{5x^2-2xy+7}{5x^2-2xy+7}$

e. $\dfrac{x^2-5x+4}{x^2-x-12}$

f. $\dfrac{x^2-25}{x^2+10x+25} \div \dfrac{x-5}{x+3}$

CL 10-153. Solve the equations below using any method. How many solutions does each problem have?

a. $\frac{6x-5}{2x+1} + \frac{2x-7}{2x+1} = 2$

b. $\sqrt{x-5} + 10 = 15$

c. $|x-7| = 22$

d. $(3x+7)^2 = 144$

CL 10-154. Solve each inequality algebraically. Then represent your solution on a number line.

a. $5x - 7 \geq 2x + 5$

b. $6x - 29 > 4x + 12$

c. $x^2 \leq -4x + 5$

d. $|2x - 7| > 31$

CL 10-155. Solve the quadratic equation below three times: once by completing the square, once by factoring and using the Zero Product Property, and once by using the Quadratic Formula. Make sure you get the same answer using each method!

$$x^2 + 14x + 40 = -5$$

CL 10-156. Graph the system of inequalities at right and shade its solutions.

$y \geq \frac{2}{3}x - 7$
$y < -x + 4$

CL 10-157. Mario and Antoine are each in the middle of reading *War and Peace*. However, they just heard that something exciting happens on page 475. Even though each boy is at a different place in the book, they each agreed to read as fast as they can and to see who can get to page 475 first. Assume they each read at a constant, but different, rate.

a. After 2 hours of reading, Mario is on page 350 and Antoine is on page 425. Who will get to page 475 first? Can you tell? **Justify** your answer.

b. After 6 hours of reading, Mario is on page 450 and Antoine is on page 465. Who will get to page 475 first? Can you tell? **Justify** your answer.

c. What page was Mario on when they started the race? What page was Antoine on when they started?

d. At what rate does Mario read? At what rate does Antoine read?

e. *War and Peace* is 1400 pages long. After how many hours will each boy finish the book?

CL 10-158. For the equation $y = \frac{5}{3}x + 7$, find:

a. The equation of the line that is parallel to the given line and passes through the point (3, 2).

b. The equation of the line that is perpendicular to the given line and passes through the point (10, 4).

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CL 10-159. Rewrite each of these expressions. Your answer should have no parentheses and no negative exponents.

    a. $4(2x^{-3}y^5)^4$

    b. $\dfrac{10x^3y^{-4}}{25x^5y^2}$

    c. $12x^{-10}y^{53}\cdot(3x^5y^{-10})^4$

    d. $\dfrac{m^2}{m^{-8}}\cdot\dfrac{3m^5}{m^9}$

CL 10-160. Check your answers to each problem above using the table at the end of the closure section. Which problems did you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

⑤   HOW AM I THINKING?

This course focuses on five different **Ways of Thinking**: reversing thinking, justifying, generalizing, making connections, and applying and extending understanding. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!
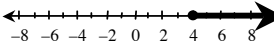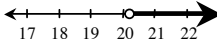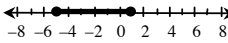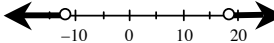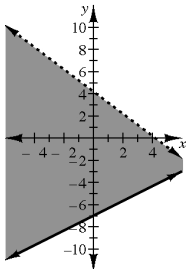
Review each of the Ways of Thinking with your class. Then choose three of these Ways of Thinking that you remember using while working in this chapter. For each Way of Thinking that you choose, show and explain where you used it and how you used it. Describe why thinking in this way helped you solve a particular problem or understand something new. (For instance, explain why you wanted to **generalize** in this particular case, or why it was useful to see these particular **connections**.) Be sure to include examples to demonstrate your thinking.

Answers and Support for Closure Activity #4
What Have I Learned?

Problem	Solution		Need Help?	More Practice
CL 10-152.	a. xy c. $x+2$ e. $\frac{x-1}{x+3}$	b. $x+2$ d. 1 f. $\frac{x+3}{x+5}$	Lessons 10.1.2 and 10.4.3 Math Notes boxes	Problems 10-3, 10-5, 10-16, 10-21, 10-97, 10-105, 10-123, and 10-128
CL 10-153.	a. $x=3.5$ c. $x=29$ or -15	b. $x=30$ d. $x=\frac{5}{3}$ or $-\frac{19}{3}$	Lessons 10.1.4, 10.2.1, and 10.2.2 Math Notes boxes	Problems 10-22, 10-49, 10-63, 10-69, 10-73, and 10-94

Problem	Solution		Need Help?	More Practice
CL 10-154.	a. $x \geq 4$		Lesson 9.2.2 Math Notes box, Lesson 10.2.4	Problems 10-9, 10-74, 10-77, 10-78, 10-80, 10-90, 10-95, 10-107, and 10-116
	b. $x > 20.5$			
	c. $-5 \leq x \leq 1$			
	d. $x > 19$ or $x < -12$			
CL 10-155.	$x = -5$ or -9		Lessons 8.1.4, 8.2.3, 8.3.1, 8.3.2, and 10.2.4 Math Notes boxes	Problems 10-106, 10-112, 10-114, and 10-139
CL 10-156.			Problems 9-57 and 9-57	Problems 10-42 and 10-86
CL 10-157.	a. Cannot be determined because we do not know which pages Mario and Antoine were on before they started racing and we do not know the rates they are reading. b. Mario and Antoine will get to page 475 at the same time. Each will arrive after another hour. c. Mario: page 300, Antoine: page 405 d. Mario: 25 pages per hour, Antoine: 10 pages per hour e. Mario: 44 hours, Antoine: 99.5 hours		Lesson 7.1.4 Math Notes box	Problems 10-23 and 10-118
CL 10-158.	a. $y = \frac{5}{3}x - 3$	b. $y = -\frac{3}{5}x + 10$	Lesson 7.3.2 Math Notes box	Problems 10-19, 10-31, 10-96, and 10-113
CL 10-159.	a. $\frac{64y^{20}}{x^{12}}$ c. $972y^{13}x^{10}$	b. $\frac{2}{5x^2y^6}$ d. $3m^6$	Lesson 10.4.3 Math Notes box	Problems 10-123, 10-128, 10-135, and 10-147